BKU

No. : 18/1/4 351362

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Signature of Invigilator

# BANKURA UNIVERSITY

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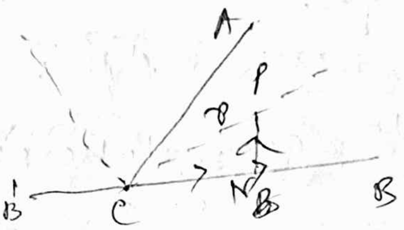
Semester .................................. Examination 20 ..................

Subject Name .................................. Subject Code

Registration Number .................................. of 20 ..................

UID Number

Equation of the bisector of the angles between two st line.




CA → a

CB → b

CN = NP. NP ∥ CB.

$r = CP = CN + NP \cdot t\frac{a}{|a|}$. ∠ACB.

$= t\cdot\frac{b}{|b|} + t\frac{a}{|a|}$

If ∠ACB' $r = t\left(\frac{b}{|b|} - \frac{a}{|a|}\right)$.

Cor: If the st lines CB and CA given by $r = c + tb$, $r = c + ta$. so that this pt of intersection is the point C whose position vector is $\vec{c}$. Then equation of bisectors are

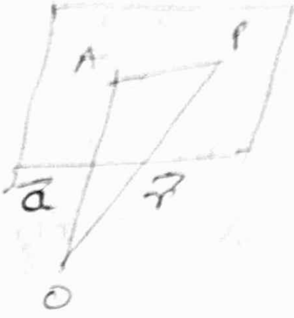
$$r = c + t\left(\frac{a}{|a|} \pm \frac{b}{|b|}\right)$$

Vector equation of a plane (Parametric form)

a) Plane through a given point and $\parallel^{l}$ to two given st lines.

$A \longrightarrow \vec{c}$ relative to origin O.

$P \to$ be any point on the required plane




The plane is $\parallel^{l}$ to the two vectors $\vec{a}$ and $\vec{b}$

$$\vec{AP} = \vec{r} - \vec{c}$$

We can take $\vec{AP} = sa + tb$

$$\boxed{r - c = sa + tb.}$$

This is the equation of plane through a given point and $\parallel^{l}$ to two given vectors.

when $c = 0$

Note:

$(r-c), a, b$ are coplanar

$(r-c) \cdot a \times b = 0 \Rightarrow [r\,a\,b] = [c\,a\,b]$

The Length of perpendicular from origin to the Plane $\dfrac{[c\,a\,b]}{|a \times b|}$

$\vec{r} = a + tb$, $\quad \vec{r} = a' + t'b'$

$$\boxed{[r\,b\,b'] = [a\,b\,b'].}$$

Plane through two given points and $\parallel^{l}$ to a given lines. $A \to a$, $B \to b$ | Vector $c$

$$\boxed{r = a + t(b-a) + sc}$$

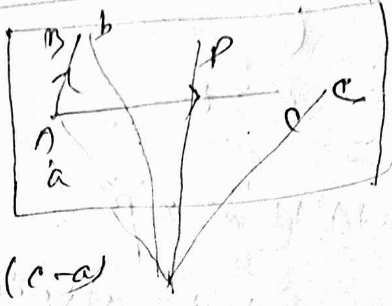
$$\boxed{(r-a)\cdot(b-a)\times c = 0}$$

c) Plane through three non collinear points

$A \to a$

$B \to b$

$C \to c$



$$r = a + s(b-a) + t(c-a)$$

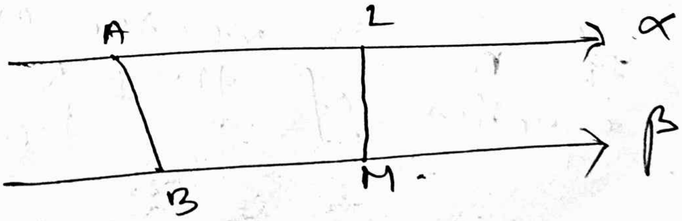
$$(r-a)\cdot(a-b)\times(b-c) = 0$$

$$r\cdot[b\times c + c\times a + a\times b] = [a\,b\,c]$$

$$\perp^{r}.\ \text{distance} = \frac{[a\,b\,c]}{|b\times c + c\times a + a\times b|}$$

Shortest Distance between two Skew lines.

Let the equ$^n$ of two skew lines $r = a + t\alpha$ and $r = b + s\beta$. through the points $A$ and $B$ respectively whose position vectors are $\vec{a}$ and $\vec{b}$.



The shortest distance between two skew lines is the distance along the line, wh in $\perp^r$ to both of them.

Let LM be the $\perp^r$ line to the lines along $\vec{\alpha}$ and $\vec{\beta}$.

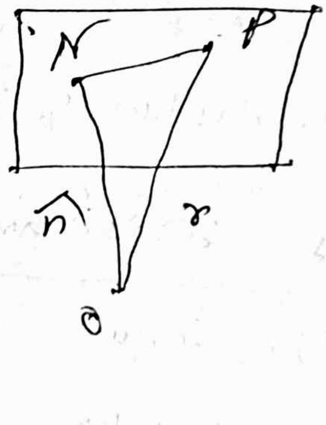
$\therefore$ Shortest distance ~~is~~ = projection of AB ~~along~~ on the st line LM.

Again $\vec{\alpha} \times \vec{\beta}$ is $\perp^r$. to both $\vec{\alpha}$ and $\vec{\beta}$.

$\therefore$ $LM \parallel \vec{\alpha} \times \vec{\beta}$.

$\therefore$ Shortest distance $= \dfrac{(a-b)\cdot(\vec{\alpha} \times \vec{\beta})}{|\alpha \times \beta|}$.

## Equation of a plane in normal form :-




r = p.v of any point P on the plane.

Let ON, is of length p, is $\perp^r$. from origin to the plane.

$\hat{n}$ = be the ~~a~~ unit vector $\perp^r$ to the plane.

Thus $\boxed{\vec{r}\cdot\hat{n} = p.}$

If n be the vector $\vec{ON}$ ~~to~~ of length n, then equation of the ~~plane~~.

$$\boxed{r.n = np = q}$$

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Cor. 1. The equation of plane passing through a point A of p.v $\vec{a}$ and perpendicular to the vector $\vec{n}$.

$$(\vec{r}-\vec{a})\cdot n = 0 \Rightarrow \boxed{\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}}$$

Cor. 2. The intercepts a, b, c made by plane $r\cdot n = q$ on the co-ordinate axes are given by

$$a i\cdot n = q \qquad b j\cdot n = q \qquad c k\cdot n = q$$

$$\boxed{a = \frac{q}{i\cdot n} \qquad b = \frac{q}{j\cdot n} \qquad c = \frac{q}{k\cdot n}}$$

Angle between two planes -

Let $\theta$ be the angle between two planes $r\cdot n_1 = q$ $\quad r\cdot n_2 = q'$

$$\Rightarrow \; n_1\cdot n_2 = |n_1|\,|n_2|\cos\theta$$

$\Rightarrow \theta = \cos^{-1} \dfrac{n_1 \cdot n_2}{|n_1|\,|n_2|}$.

## Distance of a point from a plane

Let the equation of a plane, in normal form

$$r \cdot n = p.$$

To find the $\perp^r$ distance of the plane from a point A whose p.v is $\vec{a}$ w.r.t O.

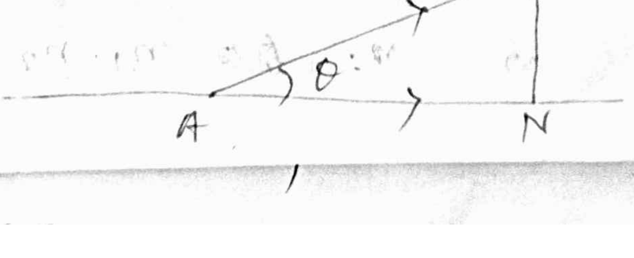
Consider a plane passing through A and parallel to the plane. We can consider as $r \cdot n = p'$ which passes through $A(\vec{a})$. i.e. $\vec{a} \cdot \vec{n} = p'$

$\therefore$ The required $\perp^r$ distance is

$$p - p' = p - a \cdot n$$

This is positive or negative according as the point A lies on the same or opposite side of the plane as the origin.

## Perpendicular distance of a point from a st. line



Let p be a point whose position vector is p and $r = a + t b$ be the equation of a given line AN passing through pt A. whose p.v. is $\vec{a}$ and parallel to $\vec{b}$.

we have to find $\perp^r$ ~~from~~ distance from p to the line AN

Let $\hat{b}$ be the unit vector along $\vec{b}$.

and $\vec{AP} = p - a$

$PN$ = Length of $\perp^r$ from p to AN

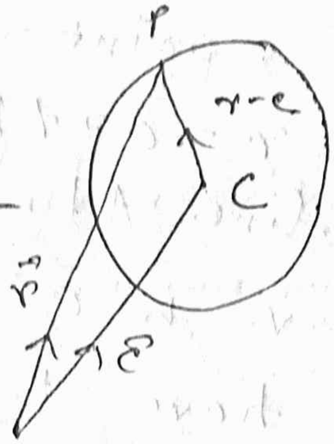
$= AP \sin\theta.$

$= |(p-a) \times \hat{b}|$

$= \dfrac{|(p-a) \times \vec{b}|}{|b|}$

## Vector equation of a Sphere :-

Let C = centre of the Sphere whose p.v. ~~of~~ $\vec{c}$ w.r.t origin.




and a be radius.

Let P be any arbitrary point the Sphere

$\vec{CP} = \vec{OP} - \vec{OC} = \vec{r} - \vec{c}$

∴ Now $|\vec{CP}|$ = radius

$\Rightarrow (r-c)^2 = a^2$

$\Rightarrow r^2 - 2r.c + c^2 = a^2$ | $K = c^2 - a^2$

$\Rightarrow \boxed{r^2 - 2r.c + K = 0}$

Cor. When O is on the ~~⊗~~ Sphere then $K = 0$

$\Rightarrow \boxed{r^2 - 2r.c = 0}$

② Equation of sphere ~~to~~ with two given points as ~~an~~ extremities of a diameter.

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Let the two given points are

$A \longrightarrow \vec{a}$ p.v.

$B \longrightarrow \vec{b}$ P.V.

Let P be any point on the Sphere whose p.v. is $\vec{r}$

$\therefore \vec{AP} = \vec{r} - \vec{a}$. $\vec{BP} = \vec{r} - \vec{b}$

Now $\angle APB = 90°$. (Semi-circle.

$(r-a)\cdot(r-b) = 0$

$\Rightarrow \boxed{r^2 - 2r\cdot(a+b) + a\cdot b = 0}$

Prob.7

## Problems

Prob. 1 Find the vector equation of plane passing through the points $(-1, 1, 2)$, $(1, -2, 1)$ and $(2, 2, 4)$.

Sol$^n$

P.V. of the three points are

$a = -i + j + 2k$

$b = i - 2j + k$

$c = 2i + 2j + 4k$.

Hence the vector equation of the required plane

$r \cdot [b \times c + c \times a + a \times b] = [a\,b\,c]$.

$\Rightarrow r \cdot (11k - 5i - 7j) = 20.$

Prob 2. Find the points, where the st lines joining the points $(3, 6, -5)$ and $(1, 2, 3)$ meets the plane passing through the points $(1, -2, 4)$, $(3, 0, 2)$ and $(3, 1, 4)$.

Sol$^n$

$r = (3i + 6j - 5k) + t\{(i + 2j + 3k) - (3i + 6j - 5k)\}$

$= (3 - 2t)i + (6 - 4t)j + (8t - 5)k.$

Again plane passing through $(1, -2, 4)$, $(3, 0, 2)$ and $(3, 1, 4)$.

$r = (1 - s - u)(i - 2j + 4k) + s(3i + 2k) + u(3i + j + 4k)$

$= (2s + 2u + 1)i + (2s + 3u - 2)j + (4 - 2s)k$

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plane.

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At the point of intersection

$3-2t = 2s + 2u + 1$

$6 - 4t = 2s + 3u - 2$

$8t - 5 = 4 - 2s$

Solving $t = \frac{19}{10}$, $s = -\frac{31}{10}$, $u = \frac{11}{5}$

$\therefore r = (3-2t)i + (6-4t)j + (8t-5)k$

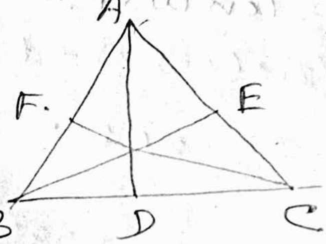
$= \left(3 - \frac{38}{10}\right) i + \left(6 - \frac{76}{10}\right) j + \left(\frac{152}{10} - 5\right) k$

$= -\frac{8}{10} i - \frac{16}{10} j + \frac{102}{10} k$

$= -\frac{4}{5} i - \frac{8}{5} j + \frac{102}{10} k$.

$= -\frac{4}{5} i - \frac{8}{5} j + \frac{51}{5} k$.

Prob. 3 Prove that the medians of triangles are concurrent and find point of concurrence.




Sol$^n$ $\Delta ABC$, D, E, F are mid pt of the sides BC, AC, AB resp.

A ⟶ $\vec{a}$ p.v.

B ⟶ p.v. $\vec{b}$

C ⟶ p.v. $\vec{c}$.

① lines $r = a + t(b+c)$ and

$\therefore D = \frac{1}{2}(b+c) \quad E = \frac{1}{2}(a+c) \quad F = \frac{1}{2}(a+b)$

Vector equation of AD and BE.

$\cancel{r = \vec{B} + t\,(1-t)\vec{b} +}$

$\vec{r} = (1-t)\vec{a} + t\cdot\frac{1}{2}(b+c)$

$\vec{r} = (1-s)\vec{b} + s\cdot\frac{1}{2}(a+c)$.

At the pt of intersection / equating the co-efficient the like vectors.

$1 - t = \frac{1}{2}s, \quad \frac{1}{2}t = 1 - s$

$\Rightarrow t = s = \frac{2}{3}$.

we get the pt of intersection $\frac{1}{3}(a+b+c)$.

Similarly we can show that the medians AD and CF intersect at $\frac{1}{2}(a+b+c)$.

$\therefore$ The medians are concurrent and pt of concurrence is $\frac{1}{3}(a+b+c)$.

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Prob. 4. The lines $L_1$ and $L_2$ have their vector eqn given by

$$\vec{r} = 3i + j + t(2j + k).$$

$$\vec{r} = 4k + s(i + j - k).$$

where s, t are scalar, Show that $L_1$ and $L_2$ intersect and find the ~~plane~~ eqn of plane containing them.

Soln We know if $L_1: r = a + tb$.

$L_2: r = a' + sb'$

Then Condition for co-planarity

$$[a\, b\, b'] = [a'\, b\, b']$$

Now $[a\, b\, b'] = (3i + j)\cdot\underbrace{(2j + k) \times (i + j - k)}_{(-3i + j - 2k)}$

$= -8.$

$[a'\, b\, b'] = 4k \cdot \underbrace{(2j + k) \times (i + j - k)}_{(-3i + j - 2k)}$.

$= -8.$

Prob. 5 Find the shortest distance two skew lines. $r = r_1 + t\alpha$ and $r = r_2 + t\beta$. where $t$ is scalar. and $r_1, \alpha, r_2, \beta$ are vector with co-ordinates $(1,-2,3)$, $(2,1,1)$, $(-2,2,-1)$ and $(-3,1,2)$ respectively.

Soln

Here $r_1 = i - 2j + 3k$. $\alpha = 2i + j + k$.

$r_2 = -2i + 2j - k$. $\beta = -3i + j + 2k$.

Now $\alpha \times \beta = (2i + j + k) \times (-3i + j + 2k)$

$= i - 7j + 5k$.

$|\alpha \times \beta| = \sqrt{1 + 49 + 25} = 5\sqrt{3}$

$r_1 - r_2 = 3i - 4j + 4k$.

$\therefore$ Shortest distance $= \dfrac{(r_1 - r_2) \cdot \alpha \times \beta}{|\alpha \times \beta|}$

$= \dfrac{(3i - 4j + 4k) \cdot (i - 7j + 5k)}{5\sqrt{3}}$

$= \dfrac{51}{5\sqrt{3}}$ unit

Prob. 6. Find the equation of a plane which contains the st line $r = t\alpha$ and is $\perp^r$ to the plane containing the st line $r = t_1\beta$ and $r = t_2\gamma$ where $t, t_1, t_2$ are scalar and $\alpha, \beta, \gamma$ are vectors.

Soln

The plane contains $r = t\alpha$ line i.e, plane passes through origin and $\parallel^l$ to $\alpha$ vector.

Again The plane is $\perp^r$. to the plane containing the ~~vector~~ lines $r = t_1\beta$ and $r = t_2\gamma$ means containing the vectors $\beta$ and $\gamma$ i.e, $\parallel^l$ to $\beta \times \gamma$ vector.

$\therefore$ plane passes through origin and $\parallel^l$ to two vector $\vec{\alpha}$, $\beta \times \gamma$ is

$$\boxed{r \cdot \alpha \times (\beta \times \gamma) = 0}$$

This is the equation of plane.

Prob. 7. Show that the st. lines $r = a + t(b+c)$ and $r = b + t(c+a)$ intersect, the point of intersection is $(a+b+c)$.

Soln. Two intersecting lines must in a same plane

$$\therefore [a \ (b+c) \ (c+a)] = [b \ (b+c) \ (c+a)]$$

Now L.H.S

$$[a \cdot (b+c) \ (c+a)]$$

$$= a \cdot (b+c) \times (c+a)$$

$$= a \cdot [b \times c + b \times a + c \times a]$$

$$= [abc].$$

R.H.S

$$[b \cdot [(b+c) \times (c+a)]]$$

$$= b \cdot [b \times c + b \times a + c \times a]$$

$$= [abc].$$

$\therefore$ These two line intersect each other.

At pt of intersection these two coinside each other. equating the co-efficient of $\vec{b}$

we have $t = 1$.

$\therefore$ point of intersection $= (a+b+c)$.

Prob. 8. Show that the st lines $r = a + t(b \times c)$ and $r = b + s(c \times a)$ may intersect if $a \cdot c = b \cdot c$.

where $a, b, c$ are three non-coplanar vectors.

Soln. These two line may intersect if $(a-b)$, $b \times c$, $c \times a$ ~~line~~ in a same plane.

$\therefore \left[(a-b)\cdot(b\times c)\times(c\times a)\right] = 0.$

$\Rightarrow \left[a\cdot b\times c \;\; c\times a\right] = \left[b \;\; b\times c \;\; c\times a\right]$

Now $(b\times c)\times(c\times a) = (b\times c)\times q \qquad q = c\times a$

$= (q\cdot b)c - (q\cdot c)b$

$= (c\times a\cdot b)c - (c\times a\cdot c)\cdot b$

$= [abc]\,c$

$\Rightarrow [a \;\; b\times c \;\; c\times a] = [abc]\,(a\cdot c).$

Again $[b\cdot b\times c \;\; c\times a] = [abc]\,(b\cdot c)$

$\therefore$ Equality holds only when $a\cdot c = b\cdot c$.

$\therefore$ These two lines may intersect if $a\cdot c = b\cdot c$

Prob. 9. Find, in terms of $K$, the shortest distance between the lines $r = \alpha + t\beta$ and $r = \gamma + t\delta$ where $\alpha = (1,2,3)$ $\beta = (2,3,4)$ $\delta = (3,4,5)$ $\gamma = (K,3,4)$ For what value of $K$ are the lines are coplanar.

Sol$^n$ Shortest distance between two lines

$$= \left| \frac{(\alpha-\gamma)\cdot \beta\times\delta}{|\beta\times\delta|} \right|.$$

~~$\alpha - \gamma = (1$.~~

~~$\gamma\ \alpha = (2,3,4) - (1,2,3)$~~

②

$\alpha - \gamma = (1,2,3) - (K,3,4)$

$= (1-K, -1, -1)$.

$$\beta \times \delta = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = i(15-16) - j(10-12) + k(8-9)$$

$$= -i + 2j - k.$$

$|\beta \times \delta| = \sqrt{1+4+1} = \sqrt{6}$.

$\therefore\ (\alpha - \gamma) \cdot \beta \times \delta = \big((1-k)i - j - k\big) \cdot (-i + 2j - k)$

~~$= -1 + 4 - 3$~~

$= (k-1) - 2 + 1 = k - 2$.

$\therefore$ Shortest distance $\dfrac{k-2}{\sqrt{6}}$.

lines are coplanar if

$(\alpha - \gamma) \cdot \beta \times \delta = 0$.

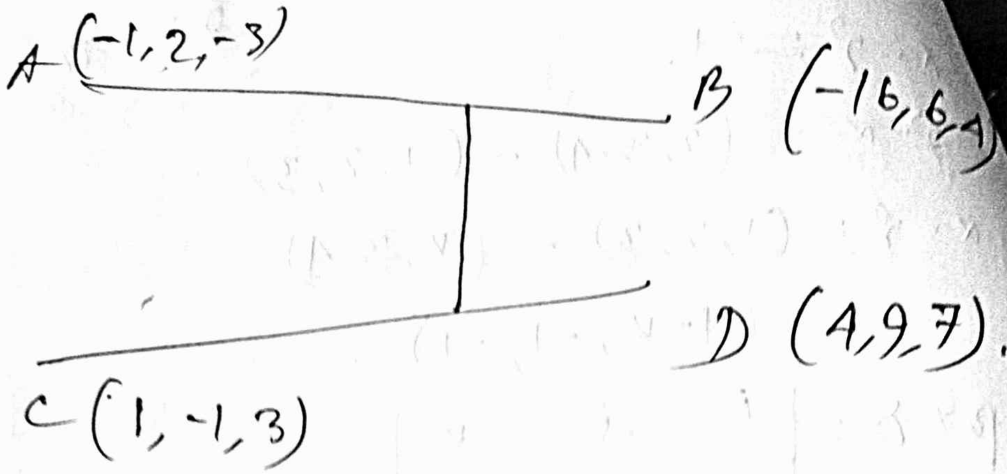
$$= \begin{vmatrix} 1-k & -1 & -1 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0 \Rightarrow (1-k)(15-16) + 1(10-12) - 1(8-9) = 0$$

$\Rightarrow k - 1 - 2 + 1 = 0$

$\Rightarrow \underline{k = 2}$

Prob 10. Calculate the shortest distance between the two st lines AB and CD, where the four points are $A = (-1, 2, -3)$ $B = (-16, 6, 4)$, $C = (1, -1, 3)$ $D = (4, 9, 7)$

Solⁿ




Here $\vec{AB} = (-16+1,\ 6-2,\ 4+3)$

$= (-15, 4, 7).$

$\vec{CD} = (4-1,\ 9+1,\ 7-3)$

$= (3, 10, 4).$

Now $\vec{AB} \times \vec{CD} = \begin{vmatrix} i & j & k \\ -15 & 4 & 7 \\ 3 & 10 & 4 \end{vmatrix}$

$= i(16-70) - j(-60-21) + k(-150-12)$

$= -54i + 81j - 162k.$

$\therefore |\vec{AB} \times \vec{CD}| = \sqrt{54^2 + 81^2 + 162^2}$

$= \sqrt{35721}$

$= 189.$

Now $\vec{AC} = (2, -3, 7)$

$\therefore SD = \left| \dfrac{\vec{AC} \cdot \vec{AB} \times \vec{CD}}{|\vec{AB} \times \vec{CD}|} \right| = \left| \dfrac{-108 - 3\times 81 - 6\times 162}{189} \right|$

$= \dfrac{1323}{189} = 7$ unit

current force.

A few number of forces acting on a point are called concurrent forces.

Let the forces $P_1$, $P_2$ ... acting on a body on a point O. is equivallent to a single force R, called the resultant.

$$R = P_1 + P_2 + P_3 + \cdots = \Sigma P_i$$ and it acts through the point O.

The system of forces will be in equillibrium if $R = 0$.

Lami's theorem

Statement If three concurrent forces be in equ^m then they are coplanar, and each is proportional to the sine of the angle between the othe two.

Proof :-

Let P, Q. R be the magnitude of the three concurrent forces and $\alpha$, $\beta$, $\gamma$ be the unit vectors along the lines of action of those forces.

Here three forces acting at a point are in equillibrium and thus closed vector polygon, length and direction whose sides are those of the forces, is a triangle. Hence the vectors are coplanar.

Since the forces are in equilibrium

$$P\alpha + Q\beta + R\gamma = 0$$

$$\Rightarrow Q\beta \times \alpha + R\gamma \times \alpha = 0$$

$$\Rightarrow Q\alpha \times \beta = R\gamma \times \alpha.$$

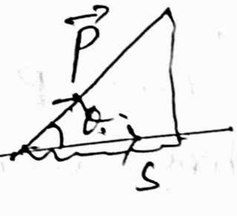
$$\Rightarrow \frac{Q}{|\gamma \times \alpha|} = \frac{R}{|\alpha \times \beta|} \Rightarrow \frac{Q}{\sin \widehat{\gamma\alpha}} = \frac{R}{\sin \widehat{\alpha\beta}}$$

Similarly $\frac{P}{|\beta \times \gamma|} = \frac{R}{|\alpha \times \beta|} \Rightarrow \frac{P}{\sin \widehat{\beta\gamma}} = \frac{R}{\sin \widehat{\alpha\beta}}$.

$$\therefore \frac{P}{\sin \widehat{\beta\gamma}} = \frac{Q}{\sin \widehat{\gamma\alpha}} = \frac{R}{\sin \widehat{\alpha\beta}}.$$

## Work done by a force

If vectors P and S represent the force and displacement respectively and $\theta$ be the angle between them, then the magnitude of the work done is given by




$$\boxed{|P|\,|S| \cos\theta = P \cdot S.}$$

No work is done when displacement is occur in ~~the~~ perpendular direction ~~of~~ to the direction of force.

11 Find the vector equation of the st line passing through a point $\vec{a}$. and $\parallel^{l}$ to the line of intersection of the planes $r \cdot n = p$ $r \cdot n' = p'$

Solⁿ $n$ and $n'$ being unit vectors along normal to the planes, $n \times n'$ is the vector $\parallel^{l}$ to the line of intersection of the planes. The required line parallel to $n \times n'$ and passing through $\vec{a}$.

$$\therefore \quad (r - \vec{a}) \cdot n \times n' = 0.$$

Prob.12 Find the point intersection of the planes $r \cdot n_1 = p_1$ $r \cdot n_2 = p_2$ $r \cdot n_3 = p_3$. where $n_1, n_2, n_3$. are three non-coplanar vector, not necessary unit vector.

Solⁿ Let $r$ be the position vector of the point intersection ~~to~~ of the planes. Since $n_1, n_2, n_3$ are non-coplanar vector, then we have $n_2 \times n_3$, $n_3 \times n_1$, $n_1 \times n_2$ are also non-coplanar vector.

Let $r = l(n_2 \times n_3) + m(n_3 \times n_1) + n(n_1 \times n_2)$ where $l, m, n$ are scalars.

Substituting $r$ in $r \cdot n_1 = p_1$

$$\Rightarrow [l.(n_2 \times n_3) + m(n_3 \times n_1) + n(n_1 \times n_2)] \cdot n_1 = p_1$$

$$\Rightarrow l[n_1 \, n_2 \, n_3] = p_1$$

$$\Rightarrow l = \frac{p_1}{[n_1 \, n_2 \, n_3]}.$$

Similarly

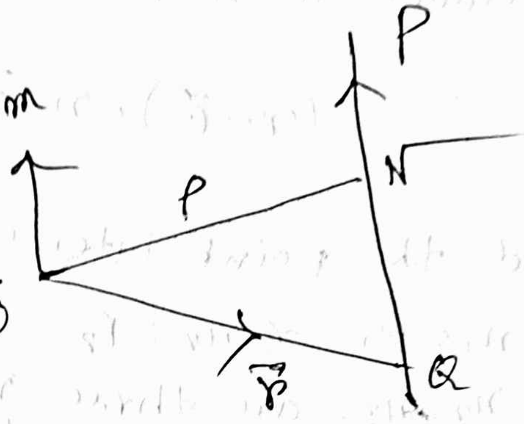
$$m = \frac{p_2}{[n_1 \, n_2 \, n_3]}$$

$$n = \frac{p_3}{[n_1 \, n_2 \, n_3]}.$$

If a number of force $P_1, P_2 \ldots P_n$ acting on a particle displace a distance $S$. then the total work done

$$P_1 \cdot S + P_2 \cdot S \cdots + P_n \cdot S = \Sigma P_i \cdot S = R \cdot S$$

## Moment of a force about a point :-




The moment of a force P about the point O is the vector $\boxed{m = r \times p}$

Where $r$ is P.V. of any point Q on the line of action of $\vec{P}$ relative to O.

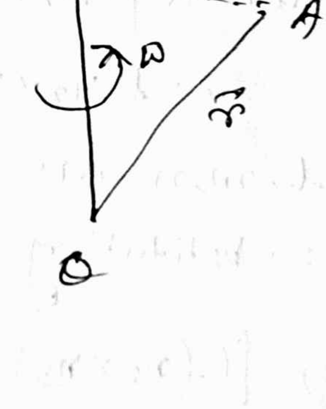
## Velocity at any point of a rotating rigid body :-

Let

$w$ = angular velocity.

OC = ~~rot~~ axis of rotation

A = fixed pt (P.V = r)




$\therefore \boxed{V = w \times r}$

The point of intersection is

$$r = \frac{1}{[n_1 n_2 n_3]}\left[P_1\, n_2 \times n_3 + P_2\, n_3 \times n_1 + P_3\, n_1 \times n_2\right]$$

Prob. 13. Find the condition that the two sphere

$r^2 - 2r\cdot c + K = 0.$ $\quad r^2 - 2r\cdot c' + K' = 0.$

may intersect orthogonally.

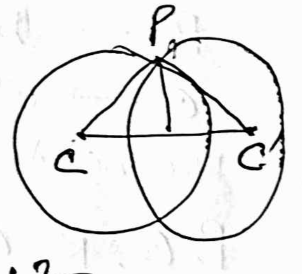
Sol$^n$:-

Let c and c' with position vector c and c' be the centres of the given spheres.

$$cc' = oc' - oc = c' - c$$

If the spheres intersect at right angle at P, then the tangent plane at P to the first sphere will pass through c' and that to the second sphere will pass through c.

Then from triangle cPc' being right angled at P. We have




$$cc'^2 = cP^2 + cP'^2$$

$$\Rightarrow (c - c')^2 = a^2 + a'^2$$

$$\Rightarrow c^2 + c'^2 - 2c\cdot c' = a^2 + a'^2$$

$$\Rightarrow c^2 - a^2 + c'^2 - a'^2 = 2c\cdot c'$$

$K = c^2 - a^2$

$K' = c'^2 - a'^2$

$$\Rightarrow \boxed{K + K' = 2cc'}$$

This is the required condition.

Prob. 14. Find the vector equation of the plane passin through $(8i + 2j - 3k)$ and perpendicular to ea of the plane.

$r \cdot (2i - j + 2k) = 0$. $r \cdot (i + 3j - 5k) + 5 = 0$

Hints

find $p = (2i - j + 2k) \times (i + 3j - 5k)$ ∴

Here $p$ is normal to the requid plane.

$(r - a) \cdot p = 0$.

Prob. 15

Forces $P$, $Q$ acted at $O$ and have a resultant $R$. If any transversal cuts their line of action at points $A, B, C$ resp. then show that

$$\frac{P}{OA} + \frac{Q}{OB} = \frac{R}{OC}$$

Sol$^n$

$A \to$ P.V of $A = a$

$B \to b$

$C \to c$

∴ The vectors representing the given forces $P, Q, R$ are $\frac{P}{OA} a$, $\frac{Q}{OB} b$ + $\frac{R}{OC} c$ resp.

$R$ is resultant of $P$ and $Q$.

$$\frac{P}{OA} a + \frac{Q}{OB} b = \frac{R}{OC} c.$$

$$\Rightarrow \frac{P}{OA} a + \frac{Q}{OB} b - \frac{R}{OC} c = 0.$$

Since $A, B, C$ are collinear, we have the sum of co-efficients in the relation is zero

$$\frac{P}{OA} + \frac{Q}{OB} - \frac{R}{OC} = 0. \Rightarrow \frac{P}{OA} + \frac{Q}{OB} = \frac{R}{OC}$$

A particle being acted on ~~a particle~~ a constant force $(4i+j-3k)$ and $(3i+j-k)$ is displaced from the point $(i+2j+3k)$ to the point $(5i+4j-k)$. Find the total work done by the forces.

Solⁿ $S$ = Displacement vector $= (5i+4j-k) - (i+2j+3k)$

$= 4i+2j-4k$.

$P$ = Total force $= (4i+j-3k) + (3i+j-k)$

$= 7i+2j-4k$.

$\therefore$ Work done $= P\cdot S = (7i+2j-4k)\cdot(4i+2j-4k)$

$= 28+4+16$

$= 48$ unit.

Prob. 17. Find the torque about the point $B(3,-1,3)$ of a force $P(4,2,1)$ passing through a point $A(5,2,4)$.

Solⁿ If $i, j, k$ are three mutually ⊥r. unit vectors. Then the position of $A$ relative to $B$ is

$r = \vec{BA} = \vec{OA} - \vec{OB} = (5i+2j+4k) - (3i-j+3k)$

$= 2i+3j+k$. and the force $P$.

$(4i+2j+k)$ ..

Hence the requird moment $= r \times P = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 4 & 2 & 1 \end{vmatrix}$

$= i+2j-8k$.